UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/51**

Paper 5 Mechanics 2 **(M2)** **October/November 2009**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

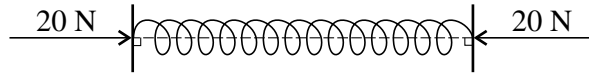At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

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[Turn over

1

$20\,\text{N} \longrightarrow$ 0000000000000000 $\longleftarrow 20\,\text{N}$

A light elastic spring of natural length 0.25 m and modulus of elasticity 100 N is held horizontally between two parallel plates. The axis of the spring is at right angles to each of the plates. The horizontal force exerted on the spring by each of the plates is 20 N (see diagram). Find the amount by which the spring is compressed and hence write down the distance between the plates.   [3]

2   A particle of mass 0.2 kg is attached to one end of a light elastic string of natural length 0.6 m and modulus of elasticity 4 N. The other end of the string is attached to a fixed point O. The particle is held at a point which is $(0.6 + x)$ m vertically below O. The particle is released from rest. In the subsequent motion the speed of the particle is $3\,\text{m s}^{-1}$ when the string becomes slack. By considering energy, find the value of x.   [5]

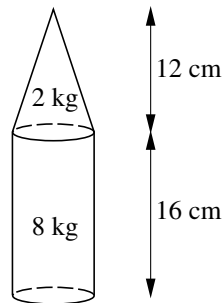**3**

12 cm

2 kg

8 kg

16 cm

Fig. 1

A uniform solid cylinder has mass 8 kg and height 16 cm. A uniform solid cone, whose base radius is the same as the radius of the cylinder, has mass 2 kg and height 12 cm. A composite solid is formed by joining the cylinder and cone so that the base of the cone coincides with one end of the cylinder (see Fig. 1).

   (i) Show that the centre of mass of the composite solid is 10.2 cm from its base.   [3]
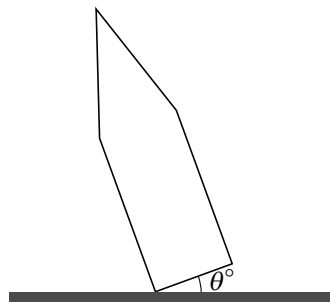

$\theta°$

Fig. 2

The composite solid is held with a point on the circumference of its base in contact with a horizontal table. The base makes an angle $\theta°$ with the table (see Fig. 2, which shows a cross-section). When the cone is released it moves towards the equilibrium position in which its base is in contact with the table.

   (ii) Given that the radius of the base is 4 cm, find the greatest possible value of θ, correct to 1 decimal place.   [3]
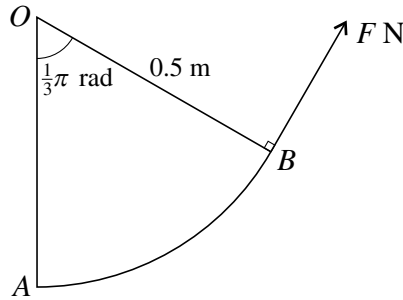
4   A particle is projected from a point O with speed $V\,\mathrm{m\,s}^{-1}$ at an angle θ above the horiz[ontal]. 0.3 s the particle is moving with speed $25\,\mathrm{m\,s}^{-1}$ at an angle $\tan^{-1}\left(\frac{7}{24}\right)$ above the horizontal.

   (i) Show that $V\cos\theta = 24$.

   (ii) Find the value of $V\sin\theta$, and hence find V and θ.   [5]

5

A uniform lamina AOB is in the shape of a sector of a circle with centre O and radius 0.5 m, and has angle $AOB = \frac{1}{3}\pi$ radians and weight 3 N. The lamina is freely hinged at O to a fixed point and is held in equilibrium with AO vertical by a force of magnitude F N acting at B. The direction of this force is at right angles to OB (see diagram). Find

   (i) the value of F,   [4]
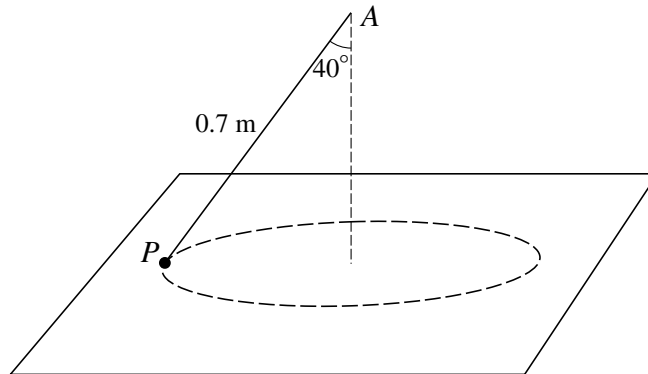
   (ii) the magnitude of the force acting on the lamina at O.   [4]

6

One end of a light inextensible string of length 0.7 m is attached to a fixed point A. The other end of the string is attached to a particle P of mass 0.25 kg. The particle P moves in a circle on a smooth horizontal table with constant speed $1.5\,\mathrm{m\,s}^{-1}$. The string is taut and makes an angle of $40°$ with the vertical (see diagram). Find

   (i) the tension in the string,   [3]

   (ii) the force exerted on P by the table.   [3]

P now moves in the same horizontal circle with constant angular speed $\omega\,\mathrm{rad\,s}^{-1}$.

   (iii) Find the maximum value of ω for which P remains on the table.   [5]

7 A particle P of mass $0.1\,\text{kg}$ is projected vertically upwards from a point O with speed 2
resistance of magnitude $0.1v\,\text{N}$ opposes the motion, where $v\,\text{m\,s}^{-1}$ is the speed of P at tim
projection.

(i) Show that, while P is moving upwards, $\dfrac{1}{v+10}\dfrac{dv}{dt} = -1$. [2]

(ii) Hence find an expression for v in terms of t, and explain why it is valid only for $0 \leqslant t \leqslant \ln 3$. [6]

(iii) Find the initial acceleration of P. [2]